

Secure wireless communication under spatial and local Gaussian noise assumptions

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Abstract

We consider wireless communication between Alice and Bob when the intermediate space between Alice and Bob is controlled by Eve. That is, our model divides the channel noise into two parts, the noise generated during the transmission and the noise generated in the detector. Eve is allowed to control the former, but is not allowed to do the latter. While the latter is assumed to be a Gaussian random variable, the former is not assumed to be a Gaussian random variable. In this situation, using backward reconciliation and the random sampling, we propose a protocol to generate secure keys between Alice and Bob under the assumption that Eve's detector has a Gaussian noise and Eve is out of Alice's neighborhood. In our protocol, the security criteria are quantitatively guaranteed even with finite block-length code based on the evaluation of error of the estimation of channel.

I. INTRODUCTION

Recently, secure wireless communication attracts much attention as a practical method to realize physical layer security [1], [2], [3], [4], [5], [6], [7], [8], [9], [10], [11], [12]. In particular, wire-tap channel model [13], [14], [15], [16] is considered as a typical model for physical layer security. In the wire-tap channel model, the authorized sender, Alice is willing to transmit her message to the authorized receiver, Bob without any information leakage to the adversary, Eve. In this case, we usually assume that the noise in the channel to Eve is larger than that in the channel to Bob. However, it is not easy to guarantee this assumption under the real wireless communication. In cryptography, it is usual to consider that the adversary, Eve is more powerful than the authorized users, Alice and Bob in some sense like RSA cryptography [17]. However, the above wire-tap channel requires the opposite assumption. So, it does not necessarily have sufficient powers of conviction to assume the above wire-tap channel in real wireless communication.

Instead of wire-tap channel model, we often employ secure key agreement, in which, Alice and Bob generate the agreed secure key from their own correlated random variables [18], [19]. This problem has a similar problem when they generate secure keys via one-way communication from Alice to Bob, because they need to assume that the mutual information between Alice and Bob is larger than that between Alice and Eve. Further, although there exist proposals to generate secure key from wireless communication [20], [21], [22], [23], [24], they do not give a quantitative security evaluation for the final keys under a reasonable assumption advantageous to Eve in a finite-length setting.

The purpose of this paper is to propose a protocol to generate quantitatively secure keys between Alice and Bob under a reasonable assumption advantageous to Eve. Since quantum key distribution [25] aims such a purpose without any assumption except for physical laws, the protocol of this paper can be regarded as an alternative of quantum key distribution under a reasonable assumption by using cheaper devices.

Further, for efficient realization of the protocol, we additionally impose the following requirements.

- (R1) The security of final keys is guaranteed quantitatively based on acceptable criterion even for cryptography community (e.g. the variational distance criterion [27] or the modified mutual information criterion) even though Eve takes the optimal strategy under the above assumption. Additionally, the formula to derive the security evaluation has sufficiently small calculation complexity.

- (R2) The calculation complexity of the whole protocol (Protocol 1 given in Section V-A) is sufficiently small.

This paper is organized as follows. Firstly, we rigorously explain our purpose and our assumption in Section II. Then, we compare our formulation with existing jamming attacks in Section III. As the next step, before proceeding to our protocol, we discuss the mathematical structure of our model in Section IV. In Section V, we give our concrete secure protocol. Section VI analyzes the security of the given protocol, and numerical demonstration of the security analysis in a typical case. Section VII is devoted to two kinds of extensions, multiple antenna attack and complex number case. Section VIII consider the relation of a model with interference to the additional noise of the eavesdropper channel so that we clarify how our model contain such a interference channel. Section IX gives proofs of statements given above.

II. PURPOSE AND ASSUMPTIONS

Recall that the aim of this paper is to propose a protocol to generate quantitatively secure keys between Alice and Bob under a reasonable assumption advantageous to Eve. Here, our aim is not to always generate secure keys, but is to detect the existence of eavesdropping with high probability when it exists. That is, when they consider there is no eavesdropper, their keys are required to be matched and secret. In other word, it is required to discard their keys when an eavesdropper exists. Here, the case without eavesdropper means the case when the operation of the eavesdropper cannot be distinguished from the natural phenomena. So, the natural case, i.e., the case with the natural phenomena, is very important in our analysis.

In the real setting, it is difficult to identify where Eve attacks the communication between Alice and Bob except for Alice's neighborhood and Bob's detector. To guarantee the security of the final keys in such a setting, it is natural to assume the following conditions to achieve the above purpose.

- (A1) The intermediate space between Alice and Bob might be controlled by Eve.
- (A2) Eve's and Bob's detectors have a Gaussian noise, and Alice and Bob know the lower bounds of the powers of their noise. This assumption is called the *local Gaussian noise assumption*. Since no detector has no detection noise, this assumption is reasonable. Nobody can control these noises.
- (A3) Alice and Bob know the lower bound of the attenuation for Alice's signal in Eve's detection. When Eve is out of Alice's neighborhood, this condition holds. This assumption is called the *spatial assumption* for Eve.
- (A4) The wireless communication between Alice and Bob is *quasi static*. That is, the channel between Alice and Bob is almost constant during a specific time interval so called the coherent time [26, Section 5.4.1]. We also assume that we can send one block of our protocol during the coherent time¹.

Therefore, when Alice sends the signal A to Bob, Bob's and Eve's detections B and E are written as

$$B := a_B A + Y + b_B X_1 + e_B, \quad (1)$$

$$E := a_E A + b_E X_2 + e_E. \quad (2)$$

Here, due to Assumptions (A2), (A3), and (A4), X_1 and X_2 are standard Gaussian random variables, Y is a random variable with average 0, and the coefficients a_B, b_B, e_B, a_E, b_E , and e_E are constants with physical meaning as Fig. I. Here, Y does not necessarily obey the Gaussian distribution. Even though we put e_E to be 0, there is no information loss. So, we consider only the case when e_E is 0 for simplicity.

Further, since the intermediate space between Alice and Bob is controlled by Eve due to Assumption (A1), the information Y can be injected by Eve as Fig. 1. Here, to discuss the situation advantageous to

¹In various protocols, a set of pulses or bits treated as one block is called a coding block. The number of such pulses or bits is called a block length. For example, in RSA cryptography, since the arithmetic is based on the public composite m , $\log m$ is a block length. Our protocol is composed of an error correcting code like an LDPC code. Since the block length of an LDPC code is from 10000 to 100000, the block length of our protocol is from 10000 to 100000 when we employ a LDPC code.

TABLE I
SUMMARY OF PARAMETERS. c_{AB} IS COVARIANCE BETWEEN A AND B . v_B IS VARIANCE OF B .

Coefficient	Meaning	Long time period behavior	Treatment in this paper	Estimation method
a_B	Attenuation	Stochastic	To be estimated by sampling	c_{AB}
a_E	Attenuation	Stochastic	Constant (upper bound among possible values)	Distance between Alice and Eve (Ass.(A3))
v_Y	Noise amplitude during transmission	Stochastic	To be estimated by sampling	$v_B - c_{AB}^2 - b_B^2$
b_B	Bob's detector noise amplitude	Constant	Constant	Performance of Bob's detector (Ass.(A2))
b_E	Eve's detector noise amplitude	Constant	Constant	Performance of Eve's detector (Ass.(A2))

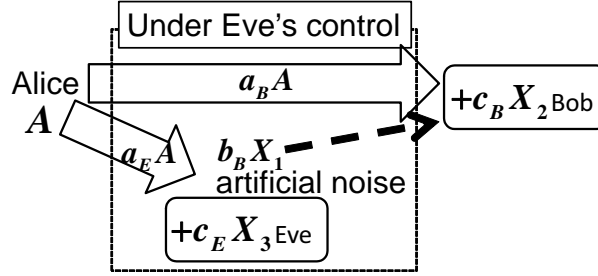


Fig. 1. Eve injects artificial noise to Bob's observation

Eve, we assume that Eve's detection has no noise except for the noise inside of her detector as Eq. (2). Thus, Eve knows the value of Y as well as E . This attack is the best strategy for Eve under Assumptions (A1)-(A4), and is called *noise injecting attack* because Eve injects the noise Y in this attack.

When Eve is closer to Alice than Bob and the performance of Eve's detector is the same as that of Bob's, the signal-noise ratio of Eve is not smaller than that of Bob so that secure communication with forward reconciliation is impossible. Since there is a possibility of secure communication with backward reconciliation, this paper considers reverse reconciliation under Assumptions (A1)-(A4). The protocol will be given in Section V.

Here, we discuss the meaning of coefficients a_B and a_E more deeply. The coefficients a_B and a_E express the attenuation. The intensities a_E^2 and a_B^2 behave as $Cd^{-\alpha}$ with positive constants C and α when the distance from Alice's transmitting antenna is d , and have stochastic behavior as fading in long time span[26, Section 5.4.1]. For example, the free space with no obstacle has the constant $\alpha = 2$ [26]. Due to *spatial assumption* (Assumption (A3)), Eve's detector is sufficiently far from Alice's transmitting antenna. So, the relation $d \geq d_0$ holds with a certain constant d_0 . Under this assumption, we can guarantee that $a_E^2 \leq Cd_0^{-\alpha}$.

On the other hand, the coefficients b_B and b_E can be lower bounded by the performance of their detectors due to Assumption (A2). As explain in Section V-A, our protocol contains random sampling. Hence, the coefficient a_B can be estimated as covariance c_{AB} between A and B in the random sampling, which provides a better estimate than the method based spatial relation between Alice and Bob. Thus, the meaning of these parameters can be summarized in Table I, while the parameter v_Y will be introduced in Section IV.

III. COMPARISON WITH OTHER ATTACKS

Here, we compare our model with the elementary jamming attack [28]. In the elementary jamming attack, Eve inserts her artificial noise to Bob's detection. However, she does not know the value of the

added noise. The purpose of jamming attack is to interrupt the communication between Alice and Bob, and is not to eavesdrop the secret information between Alice and Bob. Hence, in the jamming attack, Eve makes the artificial noise so large that the error correction by Alice and Bob does not work. That is, the jamming attack might make their final keys mismatched. Since our protocol contains the the process of estimation of channel parameters, when Eve makes the elementary jamming attack, Alice and Bob can detect such a large noise, i.e., the existence of the elementary jamming attack.

In our noise injecting attack model, Eve is allowed to know the value of the added noise. Hence, even when the artificial noise is as small as the natural case, she might obtain a part of information of the final keys. Hence, it is difficult for Alice and Bob to find the existence of the noise injecting attack. That is, Eve in our noise injecting attack model is more powerful than Eve in jamming attack. In such a scenario, Alice and Bob need to prepare their protocol so that their information transmission is secure against the most powerful Eve within the scope of their assumption.

Now, we compare our model with channel-hopping jamming attack. In channel-hopping jamming attack, Eve overrides the signal from Alice to Bob for spoofing [11], [9], [29]. However, such an attack can be prevented by the authentication between Alice and Bob. That is, our protocol is secure even against channel-hopping jamming attack by equipping authentication.

Next, we explain how our assumption covers the case when Eve can change her strategy dynamically. Alice and Bob can choose the detailed parameter of the secure key distillation protocol (Protocol 2 given in Section V-A) depending on the coding block because our secure key distillation protocol will be done as post processing. Assumption (4) means that Eve cannot change her strategy during the coherent time interval. That is, he can change her strategy only in the next coherent time interval. Since we estimate channel parameters for each coherent time interval, our protocol properly reflects such a dynamical change. In this way, our assumption is more general than existing attacks and covers various types of attacks.

IV. MATHEMATICAL STRUCTURE

Before proceeding to our protocol, we discuss the mathematical structure of our model. Firstly, we prepare the following lemma.

Theorem 1: Assume the models (1) and (2). Also, we assume that the random variables X_1 , X_2 , and A are independent standard Gaussian random variables and Y is an independent variable with average 0. Then, we have the following relation

$$P_{B|(E,Y)=(e,y)} = P_{B|E'=e'}, \quad (3)$$

where $E' := \frac{a_B a_E}{a_E^2 + b_E^2} E + Y$, $e' = \frac{a_B a_E}{a_E^2 + b_E^2} e + y$, and $P_{X|E'=e'}$ is the conditional distribution for X when E' is e' . Also, $P_{B|E'=e'}$ is the Gaussian distribution with average $e' + e_B$ and variance $v_{B|E'} := \frac{a_B^2 b_E^2}{a_E^2 + b_E^2} + b_B^2$.

This theorem implies that the noise injecting attack can be reduced to the attack only with the random variable E' .

Proof: We introduce the random variable $F := b_E A - a_E X_2$, which is a Gaussian random variable independent of E , Y , and X_1 . Its variance is $b_E^2 + a_E^2$. Since $A = \frac{a_E}{a_E^2 + b_E^2} E + \frac{b_E}{a_E^2 + b_E^2} F$, we have $B = a_B A + Y + b_B X_1 + e_B = \frac{a_B a_E}{a_E^2 + b_E^2} E + Y + \frac{a_B b_E}{a_E^2 + b_E^2} F + b_B X_1 + e_B$. Since $\frac{a_B b_E}{a_E^2 + b_E^2} F + b_B X_1$ is a Gaussian random variable with the variance $\frac{a_B^2 b_E^2}{a_E^2 + b_E^2} + b_B^2$, we obtain the desired statement. ■

To evaluate the amount of the information leaked to Eve, we need to estimate the distribution $P_{E'}$ of the random variable E' . However, we can directly estimate only the distribution $P_{A'}$ of the random variable $A^c := Y + b_B X_1 = B - a_B A - e_B$. When $\frac{a_B^2 a_E^2}{a_E^2 + b_E^2} \geq b_B^2$, we can estimate the distribution $P_{E'}$ by applying the Gaussian convolution to the distribution $P_{A'}$. However, when $\frac{a_B^2 a_E^2}{a_E^2 + b_E^2} < b_B^2$, we cannot apply this method. Indeed, we can estimate the distribution $P_{E'}$ by applying the Gaussian deconvolution, which is the inverse operation of the Gaussian convolution. But, it is quite difficult to estimate the amount of the error of our estimate of the distribution $P_{E'}$ when we employ the Gaussian deconvolution. Since

our evaluation of the amount of leaked information requires the evaluation of the amount of error in the estimation of the distribution, we employ the distribution $P_{A'}$ instead of the distribution $P_{E'}$ as follows.

In this case, we introduce two independent standard Gaussian random variables Z_1 and Z_2 so that we have another model $B = \frac{a_B a_E}{a_E^2 + b_E^2} E + Y + \frac{a_B b_E}{a_E^2 + b_E^2} F + \sqrt{b_B^2 - \frac{a_B^2 a_E^2}{a_E^2 + b_E^2}} Z_1 + \frac{a_B a_E}{\sqrt{a_E^2 + b_E^2}} Z_2 + e_B$, whose security is equivalent to the original model. So, we assume that Eve knows the random variable Z_1 as well as the random variables E and Y . In this model, similar to Theorem 1, we have

$$P_{B|(E,Y,Z_1)=(e,y,z)} = P_{B|E''=e''}, \quad (4)$$

where $E'' := \frac{a_B a_E}{a_E^2 + b_E^2} E + Y + \sqrt{b_B^2 - \frac{a_B^2 a_E^2}{a_E^2 + b_E^2}} Z_1$ and $e'' := \frac{a_B a_E}{a_E^2 + b_E^2} e + y + \sqrt{b_B^2 - \frac{a_B^2 a_E^2}{a_E^2 + b_E^2}} z$. Also, $P_{B|E''=e''}$ is the Gaussian distribution with average $e'' + e_B$ and variance a_B^2 .

Since Eve with this model is more powerful than Eve with the original model, it is enough to evaluate the security with this model. Due to (4), the security in this model can be discussed by using the distribution $P_{E''}$. Since the distribution of $b_B X_1$ is the same as that of $\frac{a_B a_E}{a_E^2 + b_E^2} E + \sqrt{b_B^2 - \frac{a_B^2 a_E^2}{a_E^2 + b_E^2}} Z_1$, the distribution of $P_{E''}$ equals to the distribution P_{A^c} . In this way, we can evaluate the amount of leaked information based on the distribution P_{A^c} .

Now, as a typical case, we assume that Y is also a Gaussian random variable with variance v_Y while it does not necessarily obey the Gaussian distribution in general. In this case, the possibility of secure key generation can be discussed by comparison of the correlation coefficient ρ_A between B and A and the correlation coefficient ρ_E between B and E . That is, when $\rho_E^2 < \rho_A^2$, we can distill secure keys from A and B with backward reconciliation. The former correlation coefficient ρ_A is calculated as

$$\rho_A^2 = \frac{a_B^2}{a_B^2 + v_Y + b_B^2}. \quad (5)$$

Since the variance of E' is $\frac{a_B^2 a_E^2}{a_E^2 + b_E^2} + b_B^2$ and the covariance between E' and B is $\frac{a_B^2 a_E^2}{a_E^2 + b_E^2} + b_B^2$, the correlation coefficient $\rho_{E'}$ between E' and B can be calculated as

$$\begin{aligned} \rho_{E'}^2 &= \frac{1}{a_B^2 + v_Y + b_B^2} \left(\frac{a_B^2 a_E^2}{a_E^2 + b_E^2} + v_Y \right) \\ &= \frac{1}{v_B} \left(\frac{c_{AB}^2 a_E^2}{a_E^2 + b_E^2} + v_B^2 - c_{AB}^2 - b_B^2 \right) \\ &= 1 - \frac{c_{AB}^2}{v_B} + \left(\frac{c_{AB}^2 a_E^2}{a_E^2 + b_E^2} - b_B^2 \right). \end{aligned} \quad (6)$$

When we find that the distribution $P_{E'}$ is sufficiently close to the Gaussian distribution, the security can be approximately evaluated by the above formula. That is, the comparison between (5) and (6) clarifies whether secure keys can be distilled. Now, we have the following lemma.

Lemma 2: The inequality $\rho_{E'}^2 < \rho_A^2$ holds if and only if

$$\frac{a_B^2}{v_Y} > \frac{a_E^2}{b_E^2} + 1. \quad (7)$$

Proof: The condition $\frac{1}{a_B^2 + v_Y + b_B^2} \left(\frac{a_B^2 a_E^2}{a_E^2 + b_E^2} + v_Y \right) < \frac{a_B^2}{a_B^2 + v_Y + b_B^2}$ is equivalent to $0 < a_B^2 - \left(\frac{a_B^2 a_E^2}{a_E^2 + b_E^2} + v_Y \right) = \frac{a_B^2 b_E^2}{a_E^2 + b_E^2} - v_Y = a_B^2 \left(\frac{b_E^2}{a_E^2 + b_E^2} - \frac{v_Y}{a_B^2} \right)$. This condition is equivalent to $1 + \frac{a_E^2}{b_E^2} = \frac{a_E^2 + b_E^2}{b_E^2} < \frac{a_B^2}{v_Y}$. So, we obtain Lemma 2. ■

However, when $\frac{a_B^2 a_E^2}{a_E^2 + b_E^2} < b_B^2$, it is possible to check whether the distribution $P_{A'}$, i.e., $P_{E''}$ is sufficiently close to the Gaussian distribution but it is not so easy to check whether the distribution $P_{E'}$ is sufficiently close to the Gaussian distribution so that we cannot employ the formula (6). Instead of (6), we calculate the correlation coefficient $\rho_{E''}$ between E'' and B can be calculated as follows. Since the variance of E''

is $v_Y + b_B^2$ and the covariance between E'' and B is $v_Y + b_B^2$, the correlation coefficient $\rho_{E''}$ between E'' and B can be calculated as

$$\begin{aligned}\rho_{E''}^2 &= \frac{1}{a_B^2 + v_Y + b_B^2} (v_Y + b_B^2) \\ &= \frac{1}{v_B} (v_B^2 - c_{AB}^2) \\ &= 1 - \frac{c_{AB}^2}{v_B}.\end{aligned}\tag{8}$$

V. PROTOCOL

A. Description of protocol

Protocol 1 Whole protocol

Alice and Bob prepare the knowledge of b_B , b_E , and a_E based on Assumptions (A2) and (A3).

STEP 1: [Initial key transmission] Alice generates her information according to standard Gaussian distribution and sends it to Bob. She repeats it $n + 2l$ times.

STEP 2: [Estimation 1] After initial communication, Alice and Bob randomly choose l samples data $(\bar{A}_1, \bar{B}_1), \dots, (\bar{A}_l, \bar{B}_l)$. They obtain the estimates \hat{e}_B , \hat{v}_B , and \hat{c}_{AB} of the average of B , the variance of B and the covariance of A and B , respectively. They also define the random variables $\bar{E}' := E' + e_B - \hat{e}_B$ and $\bar{E}'' := E'' + e_B - \hat{e}_B$.

STEP 3: [Estimation 2] Alice and Bob randomly choose another l samples data $(\tilde{A}_1, \tilde{B}_1), \dots, (\tilde{A}_l, \tilde{B}_l)$. Based on \hat{e}_B , \hat{v}_B , and \hat{c}_{AB} , they obtain the estimates \hat{P}_{A^c} and $\hat{P}_{\bar{E}'}$ of the distributions P_{A^c} and $P_{\bar{E}'}$ when $\frac{\hat{c}_{AB}^2 a_E^2}{a_E^2 + b_E^2} \geq b_B^2$. Otherwise, they obtain only the estimate \hat{P}_{A^c} of the distribution P_{A^c} , which works as the estimate $\hat{P}_{\bar{E}''}$ of $P_{\bar{E}''}$ as well.

STEP 4: [Secure key distillation] Based on the above estimates, Alice and Bob apply the backward secure key distillation protocol for n data, which will be explained as Protocol 2.

The whole protocol for noise injecting attack is given as Protocol 1, which runs Protocol 2 (backward secure key distillation protocol) as a subprotocol. Before Protocol 2, we discuss secure key distillation protocols. Although there exist several methods to asymptotically attain the optimal one way key distillation rate from Gaussian random variables by using suitable discretization [34], [35], [36], [37], there is no protocol to distill secure keys from Gaussian random variables satisfying the following conditions.

- (1) The whole calculation complexity is not so large.
- (2) A security evaluation of the final key is available with finite block-length.

Since the difficulty of its efficient construction is caused by the continuity, we employ very simple discretization in our protocol. Before describing the secure key distillation protocol, we prepare notations for hash functions. We consider a randomized function f_H from $\mathbb{F}_2^{n_1}$ to $\mathbb{F}_2^{n_2}$, where H is the random variable identifying the function f_H , and $m_1 := n_1 - n_2$ and n_2 are called the *sacrifice bit length* and the output length, respectively. Alice and Bob need to prepare random seeds H to identify the function f_H . The seeds H is allowed to be leaked to Eve. A randomized function f_H is called a universal2 hash function when the collision probability satisfies the inequality

$$\Pr\{f_H(c) = f_H(c')\} \leq 2^{n_2 - n_1}\tag{9}$$

for any distinct elements $c \neq c' \in \mathbb{F}_2^{n_1}$ [38], [39]. In the above equation, \Pr expresses the probability with respect to the choice of H . Under these preparations, we give our protocol satisfying the above conditions (1) and (2) as Protocol 2.

Thanks to the step of Error verification, we do not need to evaluate the decoding error probability in the step of Information reconciliation. That is, we do not need to care about the estimation error in the

Protocol 2 Backward secure key distillation protocol for n data

Alice and Bob prepare the estimates $\hat{P}_{E'}$ (or $\hat{P}_{E''}$), \hat{P}_{A^c} , \hat{e}_B , and \hat{c}_{AB} .

STEP 1: [Discretization] Bob converts his random variable $B - \hat{e}_B$ to 1 or -1 by taking its sign, i.e., he obtains the new bit random variable B' in \mathbb{F}_2 as $(-1)^{B'} = \text{sgn } B - \hat{e}_B$.

STEP 2: [Information reconciliation] Based on the capacity $I[\hat{P}_{A^c}, \hat{c}_{AB}]$ of the channel $W_{A|B'}$:

$$W_{A|0}(a) := \frac{\frac{1}{\sqrt{2\pi}} e^{-\frac{a^2}{2}} \int_0^\infty \hat{P}_{A^c}(b - \hat{c}_{AB}a) db}{\int_{-\infty}^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{a'^2}{2}} \int_0^\infty \hat{P}_{A^c}(b' - \hat{c}_{AB}a') db' da'},$$

$$W_{A|1}(a) := \frac{\frac{1}{\sqrt{2\pi}} e^{-\frac{a^2}{2}} \int_{-\infty}^0 \hat{P}_{A^c}(b - \hat{c}_{AB}a) db}{\int_{-\infty}^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{a'^2}{2}} \int_{-\infty}^0 \hat{P}_{A^c}(b' - \hat{c}_{AB}a') db' da'},$$

Alice and Bob prepare an error correcting code $C \subset \mathbb{F}_2^n$, whose choice will be explained latter. Bob computes the syndrome as an element $[B'^m]$ of the coset space \mathbb{F}_2^n/C from his bit sequence B'^m , calculate its representative element $\alpha([B'^m])$ in \mathbb{F}_2^n , and sends $\alpha([B'^m])$ to Alice. Bob calculates $B'^m - \alpha([B'^m]) \in C$. Alice applies the error correction to the data $((-1)^{\alpha([B'^m])_i} A_i)_{i=1}^n$ so that she obtains the estimate of $B'^m - \alpha([B'^m]) \in C$. Here, the error correction is based on the channel $\{W_{Z|0}, W_{Z|1}\}$.

STEP 3: [Privacy amplification] Based on $\hat{P}_{E'}$ or $\hat{P}_{E''}$ in addition to \hat{e}_B and \hat{c}_{AB} , Alice and Bob decide sacrifice bit length m_1 , whose choice will be explained latter. Then, they apply universal2 hash function to their bits in C with sacrifice bit length m_1 . They obtain the keys with length $\dim C - m_1$. Here, Alice (or Bob) generates the random seeds locally and can send it to Bob (or Alice) via public channel.

STEP 4: [Error verification] Alice and Bob choose the bit length m_2 for error verification. They apply another universal2 hash function to the keys with output length m_2 . They exchange their output of the universal2 hash function. If they are the same, discarding their final m_2 bits from their keys, they obtain their final keys. If they are different, they discard their keys.

step of Information reconciliation. However, we need to be careful for the estimation error in the step of Privacy amplification because no method can evaluate the amount of information leaked to Eve in the final keys without use of the estimation error.

Rigorously, in this protocol, Eve might override the signals to Bob for spoofing [11], [9]. To avoid Eve's spoofing, Alice and Bob need to authenticate each other [38], [39], [42], [43]. Alice and Bob can authenticate each other by using universal2 hash function. This authentication consumes a small number of secret keys between Alice and Bob. Since the length of the keys for the authentication is smaller than the length of generated keys, Alice and Bob can increase the length of the secret keys efficiently. When we consume k bits for the authentication for n -bit transmission, the authentication scheme is secure with a failure probability of $n2^{-k+1}$ [42, Theorem 9]. So, If Alice and/or Bob find disagreement, they consider that there exists spoofing and discard the obtained random variable. Then, this protocol well works totally.

Finally, we discuss the effects of the stochastic behaviors of the coefficients a_B and a_E due to fading. Even though this condition does not necessarily hold even in the average case with respect to this stochastic behavior, Alice and Bob might be able to efficiently generate secure keys. In this case, Alice and Bob need to assign a_E to the maximum value among possible values. On the other hand, by random sampling, they can observe whether each coding block can generate secure keys. Hence, there might be a possibility that a part of coding blocks can generate secure keys. That is, they can apply the backward secure key distillation protocol only to the coding blocks that can generate secure keys. Such a selection of advantageous events to Alice and Bob is called *post selection*.

B. Calculation complexity

Now, we discuss the calculation complexity of our protocol. In Protocol 1 except for Protocol 2, we calculate only the averages of the obtained data and its square. So, their calculation complexity is not so large. Protocol 2 contains the calculation of syndrome, the decoding of the given error correction code, and universal2 hash function. If we choose suitable code, e.g. LDPC codes, the calculation of syndrome and the decoding of the given error correction code, has not so large calculation complexity. A typical example of a universal2 hash function is given by using Toeplitz matrix. Its detail construction and the evaluation of the complexity of its construction are summarized in the recent paper [40]. Its calculation complexity is $O(m \log m)$ when m is the input length. Indeed, it was reported in paper [40] that the above type hash function practically implemented with $m = 1000000$ by a conventional personal computer. So, the parts of privacy amplification and error verification have only calculation complexity $O(n \log n)$. In error verification, we can guarantee the correctness with probability $1 - 2^{-m_2}$, which is called the significance level[41, Section VIII]. So, it is enough to choose m_2 depending on the required significance level. Here, we need to calculate the size of the sacrifice bit length in privacy amplification. This length should be chosen so that the security criterion (15) or/and (16) is less than a given threshold, which shows the security level. This calculation can be done by using the formulas (17) or/and (18), whose calculation complexity does not depend on the numbers of input and output lengths, as explained in Subsection VI. So, this process also can be done efficiently.

To achieve a larger key generation rate, we need to choose an error correction code $C \subset \mathbb{F}_2^n$ whose coding rate is close to the capacity. For this purpose, we employ an LDPC code with the brief propagation method, whose block length is around $2^{16} \cong 65,000$ [55, Chap. 4]. However, we do not necessarily choose the block length of the error correcting code to be the block length n of our protocol. That is, we can consider the concatenation of our error correcting code. When the block length of our error correcting code is $\frac{n}{k}$, k blocks of our error correcting code is treated as one block of our of our protocol, i.e., we apply one hash function to k blocks of corrected keys of error correction. That is, our LDPC code $C \subset \mathbb{F}_2^{n/k}$ is chosen so that the dimension is less than $\frac{n}{k} I[\hat{P}_{Ac}, \hat{c}_{AB}]$. Since the agreement between Alice and Bob can be checked by error correction, we do not need to evaluate the error of estimation \hat{P}_{Ac} . That is, to decide the block length n of our protocol, we need to care about only the calculation complexity of hash function.

VI. SECURITY ANALYSIS AND SACRIFICE BIT LENGTH

A. Security analysis with known parameters and distribution

For our security analysis, we introduce the random variable $\bar{E}' := E' + e_E - \hat{e}_B E' - \frac{a_B b_E}{a_E^2 + b_E^2} F - b_B X_1 - \hat{e}_B$. Since Eve knows the exchanged information via public communication, she knows \hat{e}_B . So, Eve's knowledge for Bob's random variable $B - \hat{e}_B = E' + \frac{a_B b_E}{a_E^2 + b_E^2} F + b_B X_1 + e_B - \hat{e}_B = \bar{E}' + \frac{a_B b_E}{a_E^2 + b_E^2} F$ can be reduced to \bar{E}' .

In this section, we derive general security formulas by using the true probability density function $P_{\bar{E}'}$ of the random variable \bar{E}' . For this purpose, we introduce the functions $H[P_{\bar{E}'}, v]$ and $\phi[P_{\bar{E}'}, v](t)$ as

$$\begin{aligned} H[P_{\bar{E}'}, v] &:= - \int_{-\infty}^{\infty} \left[\Phi\left(\frac{x}{\sqrt{v}}\right) \log \Phi\left(\frac{x}{\sqrt{v}}\right) dx \right. \\ &\quad \left. + (1 - \Phi\left(\frac{x}{\sqrt{v}}\right)) \log(1 - \Phi\left(\frac{x}{\sqrt{v}}\right)) \right] P_{\bar{E}'}(x) dx, \end{aligned} \quad (10)$$

and

$$\begin{aligned} \phi[P_{\bar{E}'}, v](t) &:= \log \int_{-\infty}^{\infty} \left(\Phi\left(\frac{x}{\sqrt{v}}\right)^{\frac{1}{1-t}} + (1 - \Phi\left(\frac{x}{\sqrt{v}}\right))^{\frac{1}{1-t}} \right)^{1-t} P_{\bar{E}'}(x) dx, \end{aligned} \quad (11)$$

where the base of the logarithm is chosen to be 2 in this paper.

Using the parameter v , we introduce the joint probability density function $P_{B', \bar{E}'}$ of the random variables B' and \bar{E}' as

$$P_{B', \bar{E}'}(0, x) = \Phi\left(\frac{x}{\sqrt{v}}\right)P_{\bar{E}'}(x), \quad P_{B', \bar{E}'}(1, x) = (1 - \Phi\left(\frac{x}{\sqrt{v}}\right))P_{\bar{E}'}(x). \quad (12)$$

So, the functions $H[P_{\bar{E}'}, v]$ and $\phi[P_{\bar{E}'}, v](t)$ are rewritten as

$$\begin{aligned} & H[P_{\bar{E}'}, v] \\ &= \int_{-\infty}^{\infty} P_{B', \bar{E}'}(0, x) \log \frac{P_{B', \bar{E}'}(0, x) + P_{B', \bar{E}'}(1, x)}{P_{B', \bar{E}'}(0, x)} dx \\ &+ \int_{-\infty}^{\infty} P_{B', \bar{E}'}(1, x) \log \frac{P_{B', \bar{E}'}(0, x) + P_{B', \bar{E}'}(1, x)}{P_{B', \bar{E}'}(1, x)} dx \end{aligned} \quad (13)$$

and

$$\begin{aligned} & \phi[P_{\bar{E}'}, v](t) \\ &= \log \int_{-\infty}^{\infty} (P_{B', \bar{E}'}(0, x)^{\frac{1}{1-t}} + P_{B', \bar{E}'}(1, x)^{\frac{1}{1-t}})^{1-t} dx. \end{aligned} \quad (14)$$

Here, we have $-\frac{d}{ds}\phi[P_{\bar{E}'}, v](s)|_{s=0} = H[P_{\bar{E}'}, v]$. The function $\phi[P_{\bar{E}'}, v](t)$ satisfies the following properties.

Lemma 3: The function $\phi[P_{\bar{E}'}, v](t)$ is convex for $t \in (0, 1)$.

This lemma is shown in Subsection IX-A. Since the limit $\lim_{t \rightarrow 0} \frac{-\phi[P_{\bar{E}'}, v](t)}{t}$ equals the conditional entropy, $H[P_{\bar{E}'}, v]$ is monotone decreasing for ρ .

As shown below, in the privacy amplification, we need to choose the sacrifice but rate $\frac{m_1}{n}$ is larger than $1 - H[P_{\bar{E}'}, v_{B|E'}]$, where n is the block length before information reconciliation and $v_{B|E'}$ was defined to be the variance $\frac{a_B^2 b_E^2}{a_E^2 + b_E^2} + b_B^2$. To show this fact, we make more precise analysis on the leaked information as follows. Using the relative entropy $D(P\|Q) := \sum_x P(x)(\log P(x) - \log Q(x))$ and the variational distance $d(P, Q) := \sum_x |P(x) - Q(x)|$, we adopt the conditional modified mutual information $I'(K : E|H)$ [44], [32] between Bob and Eve and the variational distance measure $d(K : E|H)$ [45] conditioned with H as

$$I'(K : E|H) := \sum_h P_H(h) D(P_{EK|H=h} \| P_{E|H=h} \times P_{U,K}) \quad (15)$$

$$d(K : E|H) := \sum_h P_H(h) d(P_{EK|H=h}, P_{E|H=h} \times P_{U,K}), \quad (16)$$

where $P_{U,K}$ is the uniform distribution for the final key. It is known that the latter satisfies the universal composable property [27].

Here, we give security formulas with true parameters. So, the discussions in [30], [31], [32], [33] yield

$$I'(K : E|H) \leq \inf_{s \in (0,1)} \frac{1}{s} 2^{s(n-m_1) + n\phi[P_{\bar{E}'}, v_{B|E'}](s)}, \quad (17)$$

$$d(K : E|H) \leq 3 \min_{t \in [0, \frac{1}{2}]} 2^{t(n-m_1) + n\phi[P_{\bar{E}'}, v_{B|E'}](t)}, \quad (18)$$

whose detail derivations are available in Subsection IX-A. Since the function $t \mapsto (n-m_1) + n\phi[P_{\bar{E}'}, v_{B|E'}](t)$ is convex (Lemma 3), the minimum $\min_{t \in [0, \frac{1}{2}]} t(n-m_1) + n\phi[P_{\bar{E}'}, v_{B|E'}](t)$ is computable by the bisection method [46, Algorithm 4.1], which gives the RHS of (18). Since $s \mapsto -\log s$ is convex, the function $s \mapsto (s(n-m_1) + n\phi[P_{\bar{E}'}, v_{B|E'}](s)) - \log s$ is convex. So, the infimum $\inf_{s \in (0,1)} t(n-m_1) + n\phi[P_{\bar{E}'}, v_{B|E'}](t)$ is computable in the same way. That is, we can calculate the RHS of (17). Since the relation holds, when the sacrifice bit length $\frac{m_1}{n}$ is greater than $1 - H[P_{\bar{E}'}, v_{B|E'}]$, there exists $s \in (0, \frac{1}{2}]$ such that $(s(1 - \frac{m_1}{n}) + \phi[P_{\bar{E}'}, v_{B|E'}](s)) < 0$. So, both upper bounds go to zero exponentially for n .

Our condition for the random hash function f_H can be relaxed to ϵ -almost universal dual hash function [47]. ([32] contains its survey with non-quantum terminology.) The latter class allows more efficient random hash functions with less random seeds [40]. Even when the random seeds H is not uniform random number, we have similar evaluations by attaching the discussion in [40]. While it is possible to apply left over hashing lemma [48], [49] and smoothing to the min entropy [45], our evaluation is better than such a combination even in the asymptotic limit, as is discussed in [30], [32].

B. Estimation with confidence level

1) *Estimation of parameters:* To estimate the average e_B , the variance v_B , the covariance c_{AB} between A and B , and the distribution $P_{E'}$, we set the confidence level $1 - \epsilon$. Due to the quasi static assumption, the random variables A , B , and E' are subject to an identical and independent distribution. Since the distribution of B is unknown, if the number l of samples is not so large, it is not easy to give the confidence interval for the estimation of e_B . However, when the number l is sufficiently large (e.g., more than 10^4), it is allowed to apply Gaussian approximation for a given confidence level $1 - \epsilon$. When the variance is unknown, we need to employ the t -distribution of degree $l - 1$. However, since the number l is sufficiently large, it can be well approximated by the Gaussian distribution. Now, we use the ϵ percent point Z_ϵ (the quantile) of the standard Gaussian distribution. The confidence interval of the average e_B is $[\hat{e}_B - \sqrt{\bar{v}_B} Z_\epsilon l^{-\frac{1}{2}}, \hat{e}_B + \sqrt{\bar{v}_B} Z_\epsilon l^{-\frac{1}{2}}]$ by using the sample mean \hat{e}_B and the unbiased variance \bar{v}_B . However, due to the largeness of l , the unbiased variance \bar{v}_B can be replaced by the sample variance because the difference is almost negligible.

Next, we estimate the variance v_B . When B is subject to the Gaussian distribution, we need to employ the χ^2 distribution of degree $l - 1$ unless the number l is sufficiently large. Now, we can apply Gaussian approximation because the number l is sufficiently large. To estimate the variance of $(B - e_B)^2$, we define the estimate $\bar{w}_B := \frac{1}{l-1} \sum_{i=1}^l ((\bar{B}_i - \hat{e}_B)^2 - \frac{1}{l-1} \sum_{i=1}^l (\bar{B}_j - \hat{e}_B)^2)$, which approximates the variance of $(B - e_B)^2$. The confidence interval of the variance v_B is $[\bar{v}_B - \sqrt{\bar{w}_B} Z_\epsilon l^{-\frac{1}{2}}, \bar{v}_B + \sqrt{\bar{w}_B} Z_\epsilon l^{-\frac{1}{2}}]$.

Now, we estimate the covariance c_{AB} between A and B . To estimate the covariance c_{AB} , we employ the sample mean \hat{c}_{AB} of $A(B - \hat{e}_B)$. To get the confidence interval of the covariance c_{AB} , we employ the unbiased variance \hat{v}_{AB} of $(A - \hat{e}_A)(B - \hat{e}_B)$, which approximates the variance of the sample mean \hat{c}_{AB} . So, the confidence interval of the covariance c_{AB} is $[\bar{c}_{AB} - \sqrt{\hat{v}_{AB}} Z_\epsilon l^{-\frac{1}{2}}, \bar{c}_{AB} + \sqrt{\hat{v}_{AB}} Z_\epsilon l^{-\frac{1}{2}}]$, where $\bar{c}_{AB} := \frac{1}{l-1} \sum_{i=1}^l (\bar{A}_i - \hat{e}_A)(\bar{B}_i - \hat{e}_B)$.

2) *Estimation of distribution:* To get the estimate $\hat{P}_{E'}$, we define the random variable $\bar{A}^c := B - \hat{c}_{AB}A - \hat{e}_B$. Using the second l data $(\bar{A}_1, \bar{B}_1), \dots, (\bar{A}_l, \bar{B}_l)$, we define our estimate

$$\hat{F}_{\bar{A}^c}(x) := \frac{1}{l} \sum_{i=1}^l I_{[-\infty, x]}(\bar{A}_i^b) \quad (19)$$

based on Kolmogorov-Smirnov test [51], [52], where $\bar{A}_i^b := \bar{B}_i - \hat{c}_{AB}\bar{A}_i - \hat{e}_B$

Define $\Phi_{AB}(x) := \Phi(\frac{x-b}{a})$. For given two distribution functions F_1 and F_2 , we define the convolution $F_1 * F_2$ as $F_1 * F_2(x) := \frac{dF_1}{dx}(x-y)F_2(y)dy$ when F_1 is differentiable. When F_2 is differentiable, the convolution $F_1 * F_2$ is defined as $F_2 * F_1$. We also define the map \mathcal{G}_{AB} as

$$\mathcal{G}_a[F] := \Phi_a * F. \quad (20)$$

When $\frac{\hat{c}_{AB}^2 a_E^2}{a_E^2 + b_E^2} \geq b_B^2$, we define our estimate as

$$\hat{F}_{E'} := \mathcal{G}_{\frac{\hat{c}_{AB}^2 a_E^2}{a_E^2 + b_E^2} - b_B^2}[\hat{F}_{\bar{A}^c}] \quad (21)$$

Otherwise, we define our estimate as $\hat{F}_{E''} := \hat{F}_{\bar{A}^c}$. The estimate $\hat{P}_{E'}$ ($\hat{P}_{E''}$) of the distribution $P_{E'}$ ($P_{E''}$) is given as the derivative of $\hat{F}_{E'}$ ($\hat{F}_{E''}$).

Now, we evaluate the error of these estimates $\hat{F}_{\bar{E}'}$ and $\hat{F}_{\bar{E}''}$. To estimate the error of this estimator, we define the Kolmogorov distribution function $L(x)$ [53];

$$L(x) := 1 - 2 \sum_{k=1}^{\infty} (-1)^{k-1} e^{-2k^2 x^2} = \frac{\sqrt{2\pi}}{x} \sum_{k=1}^{\infty} e^{-(2k-1)^2 \pi^2 / (8x^2)}, \quad (22)$$

which can also be expressed by the Jacobi theta function $\vartheta_{01}(z=0; \tau=2ix^2/\pi)$. Then, we have the following lemma, which will be shown in Subsection IX-B.

Lemma 4: Under the condition $\frac{\hat{c}_{AB}^2 a_E^2}{a_E^2 + b_E^2} \geq b_B^2$, the inequality

$$\sup_x |F_{\bar{E}'}(x) - \hat{F}_{\bar{E}'}(x)| \quad (23)$$

$$\leq \frac{\sqrt{\hat{v}_{AB}}}{\sqrt{2\pi e \hat{c}_{AB}} \sqrt{l}} Z_\epsilon + \frac{1}{\sqrt{l}} L^{-1}(1 - \epsilon) \quad (24)$$

holds with confidence level almost $1 - 2\epsilon$ when l is sufficiently large. Under the condition $\frac{\hat{c}_{AB}^2 a_E^2}{a_E^2 + b_E^2} < b_B^2$, the inequality

$$\sup_x |F_{\bar{E}''}(x) - \hat{F}_{\bar{E}''}(x)| \quad (25)$$

$$\leq \frac{\sqrt{\hat{v}_{AB}}}{\sqrt{2\pi e \hat{c}_{AB}} \sqrt{l}} Z_\epsilon + \frac{1}{\sqrt{l}} L^{-1}(1 - \epsilon) \quad (26)$$

holds with confidence level almost $1 - 2\epsilon$ when l is sufficiently large.

C. Security analysis with estimation

In our protocol given in Section V-A, in addition to the choice of hash functions, there are other random variables that are publicly transmitted. For example, the information for error estimation is publicly transmitted between Alice and Bob. So, the collection of them are denoted by C , and its distribution is denoted by P_C , which depends on the distribution P_Y and the parameters a_B and e_B . In this case, the length of sacrifice bit length m_1 depends on C . So, it is denoted by $m_1(C)$. The distribution P_H of the choice of the hash function also depends $m_1(C)$. So, it is given by the conditional distribution $P_{H|m_1(C)}$. Since the length of final keys also depends on C , the uniform distribution is given by the conditional distribution $P_{U,K|C}$. Thus, the security criteria (15) and (16) are modified to

$$I'(K : E|HC) := \sum_{c,h} P_C(c) P_{H|m_1(C)=m_1(c)}(h) D(P_{EK|H=h,C=c} \| P_{E|H=h,C=c} \times P_{U,K|C=c}) \quad (27)$$

$$d(K : E|HC) := \sum_{c,h} P_C(c) P_{H|m_1(C)=m_1(c)}(h) d(P_{EK|H=h,C=c}, P_{E|H=h,C=c} \times P_{U,K|C=c}). \quad (28)$$

For given public information C , we define

$$2^{\hat{\phi}(C,\epsilon)(t)} := \begin{cases} 2^{\phi[\hat{P}_{\bar{E}'}, \underline{v}_{B|E'}](t)} + 2(1 - 2^{-t}) \left(\frac{\sqrt{\hat{v}_{AB}}}{\sqrt{2\pi e l \hat{c}_{AB}}} Z_\epsilon + \frac{1}{\sqrt{l}} L^{-1}(1 - \epsilon) \right) & \text{when } \frac{\hat{c}_{AB}^2 a_E^2}{a_E^2 + b_E^2} \geq b_B^2 \\ 2^{\phi[\hat{P}_{\bar{E}''}, \underline{v}_{B|E'}](t)} + 2(1 - 2^{-t}) \left(\frac{\sqrt{\hat{v}_{AB}}}{\sqrt{2\pi e l \hat{c}_{AB}}} Z_\epsilon + \frac{1}{\sqrt{l}} L^{-1}(1 - \epsilon) \right) & \text{otherwise,} \end{cases} \quad (29)$$

where $\underline{v}_{B|E'} := \frac{(\hat{c}_{AB} - \frac{\sqrt{\hat{v}_{AB}}}{\sqrt{l}} Z_\epsilon)^2 b_E^2}{a_E^2 + b_E^2} + b_B^2$ and $\underline{c}_{AB} := \hat{c}_{AB} - \frac{\sqrt{\hat{v}_{AB}}}{\sqrt{l}} Z_\epsilon$.

Hence, we obtain the following lemma, which will be shown in Subsection IX-C.

Lemma 5: The inequality

$$2^{\phi[P_{\bar{E}'}, v_{B|E'}](t)} \leq 2^{\hat{\phi}(C,\epsilon)(t)} \quad (30)$$

holds with confidence level almost $1 - 2\epsilon$ when l is sufficiently large.

Combining Lemma 5 with the formulas (17) and (18), we obtain the following theorem.

Theorem 6: The inequality

$$I'(K : E|HC) \leq \inf_{s \in (0,1)} \frac{1}{s} 2^{s(n-m_1)+n\hat{\phi}(C,\epsilon)(s)}, \quad (31)$$

$$d(K : E|HC) \leq 3 \min_{s \in [0, \frac{1}{2}]} 2^{s(n-m_1)+n\hat{\phi}(C,\epsilon)(s)}, \quad (32)$$

holds with confidence level almost $1 - 2\epsilon$ when l is sufficiently large.

D. Typical case

To treat our model more concretely, we consider a typical case as follows. In this subsection, for simplicity, we discuss only the case when Y is a Gaussian random variable and $e_B = \hat{e}_B = 0$. First, we assume that there is no error between the true values and our estimations. That is, $\hat{e}_B = e_B$, $\hat{c}_{AB} = a_B$, $\hat{v}_B = a_B^2 + v_B + b_B^2$, and $\hat{P}_{Ac} = P_{Ac}$. By using the probability density function φ of the standard Gaussian variable and the correlation coefficient, the quantity $H[P_{\bar{E}}, v_{B|E'}]$ can be written to be $H[\varphi, \frac{1-\rho_E^2}{\rho_E^2}]$ because $\frac{1-\rho_E^2}{\rho_E^2} = \frac{v_{B|E'}}{v_{E'}}$. So, the required sacrifice bit rate is $1 - H[\varphi, \frac{1-\rho_E^2}{\rho_E^2}]$, which is mutual information between E' and B' . On the other hand, the mutual information $I(\hat{P}_{Ac}, \hat{c}_{AB})$ between A and B' is calculated to be $1 - H[\frac{1-\rho_A^2}{\rho_A^2}]$. That is, the secure key generation rate is $H[\varphi, \frac{1-\rho_E^2}{\rho_E^2}] - H[\varphi, \frac{1-\rho_A^2}{\rho_A^2}]$ under the reverse information reconciliation.

Now, we consider the following special case. Eve's detector has the same performance as Bob's detector, i.e., $b_E = b_B$, which will be denoted by b . The coefficients a_B and a_E for attenuations equals the same value $\sqrt{2}b$. By using the variance of the noise Y , the correlation coefficients ρ_A and ρ_E are calculated as $\rho_A^2 = \frac{2}{3+\frac{v_Y}{b^2}}$ and $\rho_E^2 = \frac{4+\frac{3v_Y}{b^2}}{9+\frac{3v_Y}{b^2}}$, i.e., $\frac{1-\rho_A^2}{\rho_A^2} = \frac{1+\frac{v_Y}{b^2}}{2}$ and $\frac{1-\rho_E^2}{\rho_E^2} = \frac{5}{4+\frac{3v_Y}{b^2}}$. If the noise Y generated in the transmission is not zero, the mutual information between A and E is larger than that between A and B . So, the forward information reconciliation cannot generate any keys. However, when we employ the reverse information reconciliation, there is a possibility to generate secure keys. When $v_Y < \frac{2b^2}{3}$, we have $\rho_A^2 > \rho_E^2$, i.e., the secure key generation rate is the positive value $H[\varphi, \frac{5}{4+\frac{3v_Y}{b^2}}] - H[\varphi, \frac{1+\frac{v_Y}{b^2}}{2}]$ under the reverse information reconciliation, which is numerically calculated as Fig. 2. In particular, when $v_Y = \frac{b^2}{5}$, the secure key generation rate $H[\varphi, \frac{5}{4+\frac{3v_Y}{b^2}}] - H[\varphi, \frac{1+\frac{v_Y}{b^2}}{2}]$ is 0.108, and the mutual informations $1 - H[\varphi, \frac{1+\frac{v_Y}{b^2}}{2}]$ and $1 - H[\varphi, \frac{5}{4+\frac{3v_Y}{b^2}}]$ are 0.372 and 0.264. In this special case, the coding rate of error correcting code needs to be less than 0.372, and the sacrifice bit rate needs to be greater than 0.264.

However, when we care about the finiteness of the block length of our code, the amount of leaked information of our final keys is not zero even though the key generation rate is less than $H[\varphi, \frac{5}{4+\frac{3v_Y}{b^2}}] - H[\varphi, \frac{1+\frac{v_Y}{b^2}}{2}]$. To discuss this issue, we need to care about estimation error. In the following, we assume the same assumption as the above discussion except for the relation between the true values and our estimations. For this purpose, we briefly discuss the upper bounds (31) and (32) by taking account into estimation error. To keep a high precision, we set the confidence level to be $1 - 10^{-4}$, i.e., $\epsilon = 5 \times 10^{-5}$. So, $Z_\epsilon = 4.06$ and $L^{-1}(1 - \epsilon) = 2.30$ [54]. For example, when we employ LDPC codes with block length $n = 1,000,000$, it is natural to choose the number of sampling to be the same value 1,000,000, i.e., $l = 500,000$. In graph 3, we numerically calculate the logarithm $s(n - m_1) + n\hat{\phi}(C, \epsilon)(s) + \log 3$ of the upper bounds appeared in (31) and (32) as a function of s when $v_Y = \frac{b^2}{5}$ and the sacrifice bit length m_1 is $0.30 \times 1,000,000 = 300,000$. The minimum value is -867 and is realized when $s = 0.07$. Here, the logarithm $s(n - m_1) + n\hat{\phi}(C, \epsilon)(s) - \log s$ of the upper bounds appeared in (31) has almost the same behavior as that in (32).

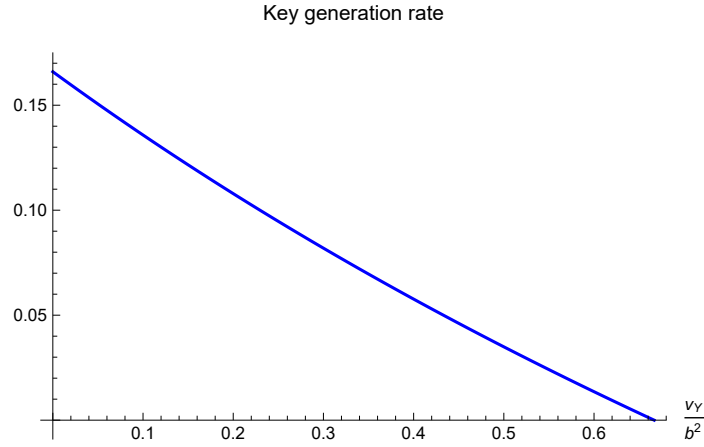


Fig. 2. Key generation rate

In this numerical calculation, we need the value \hat{v}_{AB} . When Y is a Gaussian random variable, the expectation of \hat{v}_{AB} is $2c_{AB}^2 + v_B$, which equals $2a_B^2 + (a_B^2 + b_B^2 + v_Y) = 7b^2 + v_Y$. Notice that the variance of A is 1. Also, $\frac{v_{B|E'}}{v_{E'}}$ is $(\frac{(\sqrt{2} - \frac{\sqrt{v_{AB}}}{b\sqrt{t}} Z_\epsilon)^2}{3} + 1)/(\frac{4}{3} + \frac{v_Y}{b^2}) = (\frac{(\sqrt{2} - \frac{\sqrt{7 + \frac{v_Y}{b^2}}}{\sqrt{t}} Z_\epsilon)^2}{3} + 1)/(\frac{4}{3} + \frac{v_Y}{b^2})$.

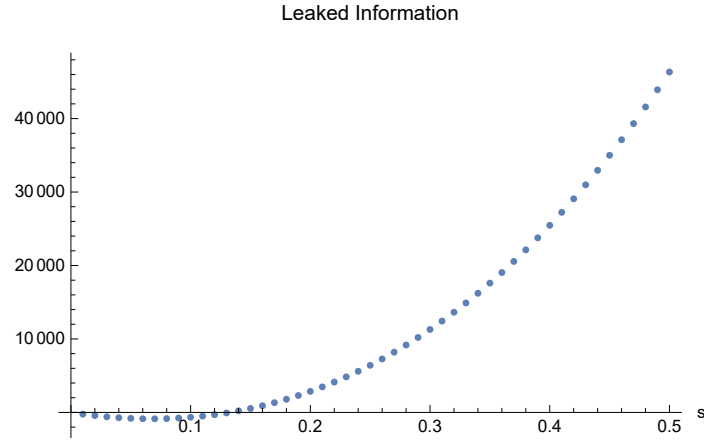


Fig. 3. Logarithm of upper bound of leaked information

VII. EXTENSIONS

A. Multi-antenna attack

As a more powerful Eve, we assume that Eve can prepare k antennas E_j ($j = 1, \dots, k$) under Assumptions (1)-(4) as

$$E_j = a_{E,j}A + b_{E,j}X_{2,j} \quad (33)$$

Bob receives B given in (1), and $X_{2,i}$ are subject to the standard Gaussian distribution independently of other random variables X_i ($i = 1, 2$).

That is, Eve knows E_1, \dots, E_k as well as Y . Now, we convert the random variables E_1, \dots, E_k to the random variable $E := \sum_{j=1}^k \frac{E_j}{a_{E,j}} = kA + \sum_{j=1}^k \frac{b_{E,j}}{a_{E,j}} X_{2,j}$ and its orthogonal complements \hat{E}_j ($j = 1, \dots, k-1$). The orthogonal complements \hat{E}_j are orthogonal to B as well as to E . Thus, all of Eve's

information for B are converted to the pair of E and Y . Therefore, we can apply Theorem 1. Notice that $\sum_{j=1}^k \frac{b_{E,j}}{a_{E,j}} X_{2,j}$ is a random variable whose variance is $\sum_{j=1}^k \left(\frac{b_{E,j}}{a_{E,j}}\right)^2$.

When all of $a_{E,j}$ and all of $b_{E,j}$ are the same values a_E and b_E , E can be written as $kA + \frac{b_E}{a_E} \sqrt{k} X_2$ by using another standard Gaussian variable X_2 . That is, E has the same information as $a_E A + \frac{b_E}{\sqrt{k}} X_2$. We can apply the above analysis with replacement of b_E by $\frac{b_E}{\sqrt{k}}$. Hence, even when there is a possibility that Eve prepares plural antennas, when Alice and Bob set the constant b_E to be a sufficiently small number, they can prevent the multi-antenna attack. When Eve prepares infinitely many antennas, Alice and Bob cannot disable Eve to access their secret information. However, considering the constraint for Eve's budget, Alice and Bob can assume a reasonable value for the constant b_E .

B. Complex number case

In the real wireless communication, all of number numbers are given as complex numbers. In this case, the random variables A , X_j , and Y are given as $A_R + iA_I$, $X_{j,R} + iX_{j,I}$, and $Y_R + iY_I$, where the random variables A_R , A_I , $X_{j,R}$ and $X_{j,I}$ are independently subject to the standard Gaussian distribution. and Y_R and Y_I are independently of other random variables. So, the noise injecting attack model with Assumptions (1)-(4) can be written as

$$B = a_B e^{i\theta_B} A + e^{i\theta_Y} Y + b_B e^{i\theta_1} X_1 + e_B e^{i\theta_3} \quad (34)$$

$$E = a_E e^{i\theta_E} A + b_E e^{i\theta_2} X_2, \quad (35)$$

where a_B, a_E, b_B , and b_E are positive real numbers. Now, we choose B_R and B_I (E_R and E_I) to be the real and imaginary parts of $e^{-i\theta_B} B$ ($e^{-i\theta_E} E$), respectively. We introduce the new random numbers $X'_{1,R}$ and $X'_{1,I}$ as the real and imaginary parts of $e^{i(\theta_1 - \theta_B)} X_1$, and $X'_{2,R}$ and $X'_{2,I}$ as those of $e^{i(\theta_2 - \theta_E)} X_2$. In the same way, we introduce Y'_R and Y'_I as the real and imaginary parts of $e^{i(\theta_Y - \theta_B)} Y$.

Then, these random numbers are also independently subject to the standard Gaussian distribution. Thus, the noise injecting attack model with real random variables can be applied as

$$B_R = a_B A_R + b_B Y'_R + b_B X'_{1,R} + e_B \cos(\theta_4 - \theta_E) \quad (36)$$

$$B_I = a_B A_I + b_B Y'_I + b_B X'_{1,I} + e_B \sin(\theta_4 - \theta_E) \quad (37)$$

$$E_R = a_E A_R + b_E X'_{2,R} \quad (38)$$

$$E_I = a_E A_I + b_E X'_{2,I}. \quad (39)$$

VIII. INTERFERENCE MODEL

In this section, to understand the model of this paper, we discuss another model, in which, Eve is weaker than Eve of the present model.

a different situation as a preparation for our analysis of noise injecting attack. That is, we replace Assumption (1) by the assumptions explained later. Due to the quasi static assumption (Assumption (4)) for both channels, Bob's and Eve's detections B and \tilde{E} are written by using Alice's signal A as

$$B := a_B A + Y + b_B X_1 + e_B, \quad (40)$$

$$\tilde{E} := a_E A + Y_2 + b_E X_2 + e_E, \quad (41)$$

where the random variables X_j are subject to the standard Gaussian distribution independently and $a_B, b_B, e_B, a_E, b_E, e_E$ are constants during the coherent time, i.e., they can be regarded for one block length for our code. The remaining random variables Y and Y_2 are independent of X_j . Also, we assume that Eve knows only all of coefficients and the random variable E , and the forms of the distributions of Y and Y_2 . Since there is no assumption for the relation between Y and Y_2 , this model contains the case with interference. These assumptions replace Assumption (A1). So, we call this assumption Assumption (A1)'. The sizes of noises are reflected in the constants b_E , and b_E . Under this model, the second terms

express the common noise during transmission, and the third terms express individual noise. Based on the local Gaussian noise assumption (Assumption (A2)), the third terms are the noises generated in their detectors.

Under Assumptions (A1)', (A2)-(A4), Alice and Bob can estimate the coefficients a_B , b_B , e_B , a_E , and b_E in the same way as in the model discussed above. Also, they can estimate the distribution of Y . However, it is impossible for them to estimate the distribution of Y_2 , i.e., type of interference during transmission. In such a case, they need to discuss the worst case.

As discussed in Section IV, when Eve is closer to Alice than Bob and the performance of Eve's detector is the same as that of Bob's, secure communication with forward reconciliation is impossible. Since there is a possibility of secure communication with backward reconciliation, this section also discusses this type of secure key generation.

Although Eve can partially obtain the information for A and B from \tilde{E} , Eve of this model can be simulated by Eve of the model in Section II because of the Markov chain $(A, B) - (Y, E) - (Y_2, E) - \tilde{E}$. Thus, even in the worst case for Alice and Bob, Eve of this model is not stronger than Eve in Section II.

However, when $Y_2 = \frac{a_E^2 + b_E^2}{a_B a_E} Y$, we have $E' = \frac{a_B a_E}{a_E^2 + b_E^2} \tilde{E}$. Due to Theorem 1, Eve's performance of this case equals that of the model in Section II. Therefore, we need to discuss the case with unknown interference, we need to consider the model given in Section II.

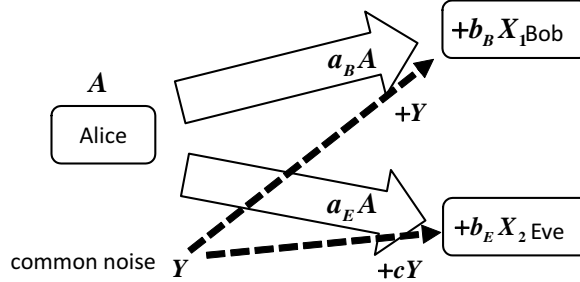


Fig. 4. Interference model: This figure shows the case when Eve has the same interference as that in Bob's detection.

IX. PROOFS

A. Proofs of (17), (18), and Lemma 3

We introduce Gallager function

$$E_0(t|P_{B'|X}, P_{B'}) := \log \int_{-\infty}^{\infty} \left(\sum_{b'} P_{B'}(b) P_{X|B'}(x|b')^{\frac{1}{1-t}} \right)^{1-t} dx,$$

which is known to be convex for t [50]. Since $2^{\phi[P_{\tilde{E}'}, v](t)} = 2^{E_0(s|P_{B'|X}, P_{B'})} \left(\left(\frac{1}{2} \right)^{1-\frac{1}{1-t}} \right)^{1-t}$, we have $\phi[P_{\tilde{E}'}, v](t) = E_0(t|P_{B'|X}, P_{B'}) + t$, which shows that $\phi[P_{\tilde{E}'}, v](t)$ is convex for t .

Now, we show (17) and (18). For this purpose, we introduce a function for a joint distribution $P_{X,Y}$ as $\phi(t|X|Y|P_{X,Y}) := \int_Y \left(\sum_x P_{X,Y}(x, y)^{\frac{1}{1-t}} \right)^{1-t} dy$, which is denoted by $-tH_{\frac{1}{1-t}}^{\uparrow}(X|Y|P_{X,Y})$ in [32] or $-tH_{\frac{1}{1-t}}^G(X|Y|P_{X,Y})$ in [33]. In this proof, we employ the rates $R_1 := \frac{\dim C}{n}$ and $R_2 := \frac{m_1}{n}$. The function

$\phi(t|X|Y|P_{X,Y})$ is a generalization of $\phi[P_{\bar{E}'}, v](t)$. Applying [30, (67)] and [33, (21)], we have

$$\begin{aligned}
& d(K : E|H) \\
& \stackrel{(a)}{\leq} 3 \min_{t \in [0, \frac{1}{2}]} 2^{tn(R_1 - R_2) + \phi(t|B'^n|[B'^n], E^n|P_{B'^n, E^n})} \\
& \stackrel{(b)}{\leq} 3 \min_{t \in [0, \frac{1}{2}]} 2^{tn(R_1 - R_2) + tn(1 - R_1)} 2^{\phi(t|B'^n|[B'^n]|E^n|P_{B'^n, E^n})} \\
& = 3 \min_{t \in [0, \frac{1}{2}]} 2^{tn(1 - R_2)} 2^{n\phi(t|B'|E|P_{B', E})} \\
& = 3 \min_{t \in [0, \frac{1}{2}]} 2^{tn(1 - R_2)} 2^{n\phi[P_{\bar{E}'}, v](t)}, \tag{42}
\end{aligned}$$

where (a) and (b) follows from [30, (67)] and [33, (21)], respectively. So, we obtain (18). When we replace the role of [30, (67)] by [32, (54) and Lemma 22], we obtain a similar evaluation as (18) for ϵ -almost universal dual hash function.

Now, we introduce another function for $P_{X,Y}$ as

$$H_{1+s}(X|Y|P_{X,Y}) := -\frac{1}{s} \log \int_{\mathcal{Y}} \left(\sum_x P_{X|Y}(x|y)^{1+s} \right) P_Y(y) dy.$$

We denote $sH_{1+s}(X|Y|P_{X,Y})$ by $\tilde{H}_{1+s}(X|Y|P_{X,Y})$ in [31] or $sH_{1+s}^\downarrow(X|Y|P_{X,Y})$ in [32]. Applying [31, (3)], [32, Lemma 5], and [33, (21)], we have

$$\begin{aligned}
& I'(K : E|H) \\
& \stackrel{(a)}{\leq} \inf_{s \in (0,1)} \frac{1}{s} 2^{sn(R_1 - R_2) - sH_{1+s}(B'^n|[B'^n], E^n|P_{B'^n, E^n})} \\
& \stackrel{(b)}{\leq} \inf_{s \in (0,1)} \frac{1}{s} 2^{sn(R_1 - R_2) + sn(1 - R_1)} 2^{\phi(s|B'^n|[B'^n]|E^n|P_{B'^n, E^n})} \\
& \stackrel{(c)}{\leq} \inf_{s \in (0,1)} \frac{1}{s} 2^{sn(R_1 - R_2) + sn(1 - R_1)} 2^{\phi(s|B'^n|[B'^n]|E^n|P_{B'^n, E^n})} \\
& = \inf_{s \in (0,1)} \frac{1}{s} 2^{sn(1 - R_2)} 2^{n\phi(s|B'|E|P_{B', E})} \\
& = \inf_{s \in (0,1)} \frac{1}{s} 2^{sn(1 - R_2)} 2^{n\phi[P_{\bar{E}'}, v](s)}, \tag{43}
\end{aligned}$$

where (a), (b), and (c) follow from [31, (3)], [32, Lemma 5], and [33, (21)], respectively. So, we obtain (17). When we need an evaluation with ϵ -almost universal dual hash function, it is sufficient to replace the role of [31, (3)] by [32, (56) and Theorem 23].

B. Proof of Lemma 4

To show Lemma 4, we employ Kolmogorov-Smirnov test[51], [52], whose detail is the following. We consider the independent random variables $\bar{X}_1, \dots, \bar{X}_l$ subject to the distribution P_X , whose cumulative distribution function is $F_X(x)$. Then, we define the empirical distribution function

$$F_{X,l}(x) := \frac{1}{l} \sum_{i=1}^l I_{[-\infty, x]}(\bar{X}_i), \tag{44}$$

where $I_{[-\infty, x]}(X)$ is the indicator function, equal to 1 if $X \leq x$ and equal to 0 otherwise. We define the random variable

$$D_{X,l} := \sup_x |F_{X,l}(x) - F_X(x)|. \tag{45}$$

Then, we have the following lemma.

Proposition 7 (Kolmogorov-Smirnov test[53]): The equation

$$\lim_{l \rightarrow \infty} \Pr(D_{X,l} \leq \frac{1}{\sqrt{l}} L^{-1}(1 - \epsilon)) = 1 - \epsilon \quad (46)$$

holds. That is, we can say that $D_{X,l} \leq \frac{1}{\sqrt{l}} L^{-1}(1 - \epsilon)$ with confidence level almost $1 - \epsilon$ when l is sufficiently large.

Thus, the relation

$$\sup_x |F_{\bar{A}^c}(x) - \hat{F}_{\bar{A}^c}(x)| \leq \frac{1}{\sqrt{l}} L^{-1}(1 - \epsilon) \quad (47)$$

holds with confidence level almost $1 - \epsilon$ when l is sufficiently large.

Next, we prepare the following two lemmas

Lemma 8: Three distribution functions F_1 , F_2 , and F_3 satisfy

$$\sup_x |F_1 * F_2(x) - F_1 * F_3(x)| \leq \sup_x |F_2(x) - F_3(x)|. \quad (48)$$

Lemma 9: When $a < 1$,

$$\sup_x |\Phi_1(x) - \Phi_a(x)| = \int_{a\sqrt{\frac{2\log a}{1-a^2}}}^{\sqrt{\frac{2\log a}{1-a^2}}} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx. \quad (49)$$

When $a \geq 1$,

$$\sup_x |\Phi_1(x) - \Phi_a(x)| = \int_{\sqrt{\frac{2\log a}{1-a^2}}}^{a\sqrt{\frac{2\log a}{1-a^2}}} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx. \quad (50)$$

When a is close to 1,

$$\sup_x |\Phi_1(x) - \Phi_a(x)| = \frac{|a-1|}{\sqrt{2\pi e}} + O((a-1)^2). \quad (51)$$

Proof: The derivative of $\Phi_1(x) - \Phi_a(x)$ with respect to x is $\frac{1}{\sqrt{2\pi}}(e^{-\frac{x^2}{2}} - \frac{1}{a}e^{-\frac{x^2}{2a^2}})$. It equals zero if and only if $x = \pm a\sqrt{\frac{-2\log a}{1-a^2}}$. Due to the symmetry $\Phi_1(x) - \Phi_a(x) = -\Phi_1(-x) - \Phi_a(-x)$, the maximum of the absolute value $|\Phi_1(x) - \Phi_a(x)|$ is realized when $x = \pm a\sqrt{\frac{-2\log a}{1-a^2}}$. Substituting this value, we obtain (49) and (50).

When a is close to 1, $\frac{-2\log a}{1-a^2} = \frac{-2\log(1+a-1)}{1-a^2} = \frac{-2((a-1)+O((a-1)^2))}{1-a^2} = \frac{2}{a+1} + O(a-1) = 1 + O(a-1)$. So, we have $e^{-\frac{-2\log a}{1-a^2}} = e^{-\frac{1}{2}} + O(a-1)$, $e^{-\frac{-2a\log a}{1-a^2}} = e^{-\frac{1}{2}} + O(a-1)$, and $\sqrt{\frac{-2\log a}{1-a^2}} - a\sqrt{\frac{-2\log a}{1-a^2}} = (1-a) + O((a-1)^2)$. Thus,

$$\int_{a\sqrt{\frac{2\log a}{1-a^2}}}^{\sqrt{\frac{2\log a}{1-a^2}}} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx = |(a-1) + O((a-1)^2)| \cdot \frac{1}{\sqrt{2\pi}} (e^{-\frac{1}{2}} + O(a-1)) = \frac{|a-1|}{\sqrt{2\pi e}} + O((a-1)^2), \quad (52)$$

which implies (51). ■

Now, we show (24) under the condition $\frac{\hat{c}_{AB}^2 a_E^2}{a_E^2 + b_E^2} \leq b_B^2$. We notice that

$$\begin{aligned} & \sup_x |F_{\bar{E}'}(x) - \hat{F}_{\bar{E}'}(x)| \\ & \leq \sup_x |F_{\bar{E}'}(x) - \Phi \sqrt{\frac{\hat{c}_{AB}^2 a_E^2}{a_E^2 + b_E^2} - b_B^2} * F_{\bar{A}^c}(x)| \\ & \quad + \sup_x |\Phi \sqrt{\frac{\hat{c}_{AB}^2 a_E^2}{a_E^2 + b_E^2} - b_B^2} * F_{\bar{A}^c}(x) - \hat{F}_{\bar{E}'}(x)|. \end{aligned} \quad (53)$$

We discuss the first term. Since

$$\begin{aligned} & \Phi \sqrt{\frac{\hat{c}_{AB}^2 a_E^2}{a_E^2 + b_E^2} - b_B^2} * F_{\bar{A}^c}(x) = \Phi \sqrt{\frac{\hat{c}_{AB}^2 a_E^2}{a_E^2 + b_E^2} - b_B^2} * F_{B - \hat{c}_{AB}A - \hat{e}_B}(x) \\ & = \Phi \sqrt{\frac{\hat{c}_{AB}^2 a_E^2}{a_E^2 + b_E^2} - b_B^2} * F_{Y + b_B X_1 + a_B A + e_B - \hat{c}_{AB}A - \hat{e}_B}(x) = \Phi \sqrt{\frac{\hat{c}_{AB}^2 a_E^2}{a_E^2 + b_E^2} - b_B^2} * F_{b_B X_1 + (a_B - \hat{c}_{AB})A + Y + e_B - \hat{e}_B}(x) \\ & = \Phi \sqrt{\frac{\hat{c}_{AB}^2 a_E^2}{a_E^2 + b_E^2} - b_B^2} * (\Phi \sqrt{b_B^2 + (a_B - \hat{c}_{AB})^2} * F_{Y + e_B - \hat{e}_B})(x) = \Phi \sqrt{\frac{\hat{c}_{AB}^2 a_E^2}{a_E^2 + b_E^2} + (a_B - \hat{c}_{AB})^2} * F_{Y + e_B - \hat{e}_B}(x), \end{aligned} \quad (54)$$

we have

$$\begin{aligned} & \sup_x |F_{\bar{E}'}(x) - \Phi \sqrt{\frac{\hat{c}_{AB}^2 a_E^2}{a_E^2 + b_E^2} - b_B^2} * F_{\bar{A}^c}(x)| \\ & = \sup_x |\Phi \frac{a_B a_E}{\sqrt{a_E^2 + b_E^2}} * F_{Y + e_B - \hat{e}_B}(x) - \Phi \sqrt{\frac{\hat{c}_{AB}^2 a_E^2}{a_E^2 + b_E^2} + (a_B - \hat{c}_{AB})^2} * F_{Y + e_B - \hat{e}_B}(x)| \\ & \stackrel{(a)}{\leq} \sup_x |\Phi \frac{a_B a_E}{\sqrt{a_E^2 + b_E^2}}(x) - \Phi \sqrt{\frac{\hat{c}_{AB}^2 a_E^2}{a_E^2 + b_E^2} + (a_B - \hat{c}_{AB})^2}(x)| \\ & \stackrel{(b)}{=} \frac{1}{\sqrt{2\pi e}} |\frac{\hat{c}_{AB}}{a_B} - 1|^2 + O((\frac{\hat{c}_{AB}}{a_B} - 1)^2), \end{aligned} \quad (55)$$

where (a) follows from Lemma 8 and (b) does from the combination of (51) and the following derivation;

$$\begin{aligned} & \frac{\sqrt{\frac{\hat{c}_{AB}^2 a_E^2}{a_E^2 + b_E^2} + (a_B - \hat{c}_{AB})^2}}{\frac{a_B a_E}{\sqrt{a_E^2 + b_E^2}}} \\ & = \sqrt{\frac{\hat{c}_{AB}^2}{a_B^2} + \frac{a_E^2 + b_E^2}{a_E^2} (1 - \frac{\hat{c}_{AB}}{a_B})^2} \\ & = \frac{\hat{c}_{AB}}{a_B} \sqrt{1 + \frac{a_E^2 + b_E^2}{a_E^2} (\frac{a_B}{\hat{c}_{AB}} - 1)^2} \\ & = \frac{\hat{c}_{AB}}{a_B} (1 + \frac{a_E^2 + b_E^2}{2a_E^2} (\frac{a_B}{\hat{c}_{AB}} - 1)^2 + O((\frac{\hat{c}_{AB}}{a_B} - 1)^4)) \\ & = \frac{\hat{c}_{AB}}{a_B} + \frac{a_E^2 + b_E^2}{2a_E^2} \frac{\hat{c}_{AB}}{a_B} (\frac{a_B}{\hat{c}_{AB}} - 1)^2 + O((\frac{\hat{c}_{AB}}{a_B} - 1)^4) \\ & = \frac{\hat{c}_{AB}}{a_B} + O((\frac{\hat{c}_{AB}}{a_B} - 1)^2). \end{aligned} \quad (56)$$

When l is sufficiently large, with confidence level almost $1 - \epsilon$, since the relation $|\frac{\hat{c}_{AB}}{a_B} - 1| \leq \frac{\sqrt{\hat{v}_{AB}}}{\hat{c}_{AB}\sqrt{l}} Z_\epsilon$ holds, the first term is upper bounded by $\frac{1}{\sqrt{2\pi e}} \cdot \frac{\sqrt{\hat{v}_{AB}}}{\hat{c}_{AB}\sqrt{l}} Z_\epsilon$.

The second term is evaluated as

$$\begin{aligned}
& \sup_x |\Phi_{\sqrt{\frac{\hat{c}_{AB}^2 a_E^2}{a_E^2 + b_E^2} - b_B^2}} * F_{\bar{A}^c}(x) - \hat{F}_{\bar{E}'}(x)| \\
&= \sup_x |\Phi_{\sqrt{\frac{\hat{c}_{AB}^2 a_E^2}{a_E^2 + b_E^2} - b_B^2}} * F_{\bar{A}^c}(x) \Phi_{\sqrt{\frac{\hat{c}_{AB}^2 a_E^2}{a_E^2 + b_E^2} - b_B^2}} * \hat{F}_{\bar{A}^c}(x)| \\
&\stackrel{(a)}{\leq} \sup_x |F_{\bar{A}^c}(x) - \hat{F}_{\bar{A}^c}(x)|,
\end{aligned} \tag{57}$$

where (a) follows from Lemma 8. So, when l is sufficiently large, the second term is upper bounded by $\frac{1}{\sqrt{l}} L^{-1}(1 - \epsilon)$ with confidence level almost $1 - \epsilon$.

Combining these two discussion, we obtain (24) with confidence level almost $1 - 2\epsilon$.

Since the relation $|\frac{\hat{c}_{AB}}{a_B} - 1| \leq \frac{\sqrt{\hat{c}_{AB}}}{\hat{c}_{AB}\sqrt{l}} Z_\epsilon$ holds with confidence level almost $1 - \epsilon$ when l is sufficiently large, combining (47), we obtain (24).

Now, we show (26). We notice that

$$\begin{aligned}
& \sup_x |F_{\bar{E}''}(x) - \hat{F}_{\bar{E}''}(x)| \\
&\leq \sup_x |F_{\bar{E}''}(x) - F_{\bar{A}^c}(x)| \\
&\quad + \sup_x |F_{\bar{A}^c}(x) - \hat{F}_{\bar{A}^c}(x)|.
\end{aligned} \tag{58}$$

Since

$$F_{\bar{E}''} = F_{E''+e_B-\hat{e}_B} = F_{A^c+e_B-\hat{e}_B} = F_{B-a_B A-e_B+e_B-\hat{e}_B} = F_{B-a_B A-\hat{e}_B} = \Phi_{a_B} * F_{B-\hat{e}_B} \tag{59}$$

and

$$F_{\bar{A}^c} = F_{B-\hat{c}_{AB} A-\hat{e}_B} = \Phi_{\hat{c}_{AB}} * F_{B-\hat{e}_B}, \tag{60}$$

the first term is evaluated as

$$\begin{aligned}
& \sup_x |F_{\bar{E}''}(x) - F_{\bar{A}^c}(x)| = \sup_x |\Phi_{a_B} * F_{B-\hat{e}_B}(x) - \Phi_{\hat{c}_{AB}} * F_{B-\hat{e}_B}(x)| \\
&\stackrel{(a)}{\leq} \sup_x |\Phi_{a_B}(x) - \Phi_{\hat{c}_{AB}}(x)| \\
&\stackrel{(b)}{=} \frac{1}{\sqrt{2\pi\epsilon}} \left| \frac{\hat{c}_{AB}}{a_B} - 1 \right|^2 + O\left(\left(\frac{\hat{c}_{AB}}{a_B} - 1\right)^2\right),
\end{aligned} \tag{61}$$

where (a) and (b) follow from Lemma 8 and (51), respectively. Thus, in the same way, we obtain (26).

C. Proof of Lemma 5

To show Lemma 5, we prepare the following lemmas.

Lemma 10: The function $v \mapsto (\Phi(\frac{x}{\sqrt{v}})^{\frac{1}{1-t}} + (1 - \Phi(\frac{x}{\sqrt{v}}))^{\frac{1}{1-t}})^{1-t}$ is monotone decreasing for any x .

Proof: The function $p \mapsto p^{\frac{1}{1-t}} + (1-p)^{\frac{1}{1-t}}$ is monotone decreasing in $[0, \frac{1}{2}]$ and is monotone increasing in $[\frac{1}{2}, 1]$. The function $v \mapsto \Phi(\frac{x}{\sqrt{v}})$ is monotone increasing in $[0, 1]$ for $x > 0$ and is monotone decreasing in $[0, 1]$ for $x < 0$. Since $\Phi(\frac{1}{\sqrt{v}}0) = \frac{1}{2}$, we conclude that the function $v \mapsto (\Phi(\frac{x}{\sqrt{v}})^{\frac{1}{1-t}} + (1 - \Phi(\frac{x}{\sqrt{v}}))^{\frac{1}{1-t}})^{1-t}$ is monotone increasing for any x . \blacksquare

Lemma 11: When a non-negative valued function f is monotone decreasing in $(-\infty, 0]$ and is monotone increasing in $[0, \infty)$, we have

$$\begin{aligned} & \left| \int_{-\infty}^{\infty} f(x) P_X(dx) - \int_{-\infty}^{\infty} f(x) P_{X'}(dx) \right| \\ & \leq 2 \sup_x (f(x) - f(0)) \sup_x |F_X(x) - F_{X'}(x)|. \end{aligned} \quad (62)$$

Lemma 11 will be shown in Subsection IX-D.

When l is sufficiently large, Lemma 4 and its proof guarantee (24) and $|\frac{\hat{c}_{AB}}{a_B} - 1| \leq \frac{\sqrt{\hat{v}_{AB}}}{\hat{c}_{AB}\sqrt{l}} Z_\epsilon$ with confidence level almost $1 - 2\epsilon$. In the following discussion, we give a statement with confidence level almost $1 - 2\epsilon$. Thus, Lemma 10 guarantees that

$$2^{\phi[P_{\bar{E}', v_{B|E'}}](t)} \leq 2^{\phi[P_{\bar{E}', \mathcal{U}_{B|E'}}](t)} \quad (63)$$

Now, we apply Lemma 11 to the case when $f(x) = (\Phi(\frac{x}{\frac{\mathcal{U}_{B|E'}}{1-t}}) + (1 - \Phi(\frac{x}{\frac{\mathcal{U}_{B|E'}}{1-t}}))^{1-t})^{1-t}$, which satisfies the condition of Lemma 11. The supremum $\sup_x (f(x) - f(0))$ is calculated as

$$\sup_x (f(x) - f(0)) = 1 - \left(\frac{2}{2^{\frac{1}{1-t}}} \right)^{1-t} = 1 - 2^{-t}. \quad (64)$$

Thus, we have

$$\begin{aligned} & |2^{\phi[P_{\bar{E}', \mathcal{U}_{B|E'}}](t)} - 2^{\phi[\hat{P}_{\bar{E}', \mathcal{U}_{B|E'}}](t)}| \\ & \leq 2 \sup_x (f(x) - f(0)) \sup_x |F_{\bar{E}'}(x) - \hat{F}_{\bar{E}'}(x)| = 2(1 - 2^{-t}) \sup_x |F_{\bar{E}'}(x) - \hat{F}_{\bar{E}'}(x)| \\ & \leq 2(1 - 2^{-t}) \left(\frac{\sqrt{\hat{v}_{AB}}}{\sqrt{2\pi e \hat{c}_{AB} \sqrt{l}}} Z_\epsilon + \frac{1}{\sqrt{l}} L^{-1}(1 - \epsilon) \right). \end{aligned} \quad (65)$$

So, combining (64) and (65), we obtain (30) when $\frac{\hat{c}_{AB}^2 a_E^2}{a_E^2 + b_E^2} \geq b_B^2$.

Using the discussion of the proof of the above case, we can show (30) when $\frac{\hat{c}_{AB}^2 a_E^2}{a_E^2 + b_E^2} < b_B^2$.

D. Proof of Lemma 11

To show Lemma 11, we prepare the following lemma.

Lemma 12: Let f be a continuous function defined on $[a, b]$, and G be a bounded function defined on $[a, b]$. We assume that G is differentiable except for a finite number of discontinuous points.

(1) When the function f is monotone decreasing in $[a, b]$ and the function G satisfies

$$\max_{y \in [a, b]} (G(y) - G(a)) = G(b) - G(a), \quad (66)$$

we have

$$\int_a^b f(x) \frac{dG}{dx}(x) dx \leq f(a) \int_a^b \frac{dG}{dx}(x) dx. \quad (67)$$

(2) When the function f is monotone increasing in $[a, b]$ and the function G satisfies

$$\min_{y \in [a, b]} (G(y) - G(a)) = G(b) - G(a), \quad (68)$$

we have

$$\int_a^b f(x) \frac{dG}{dx}(x) dx \leq f(a) \int_a^b \frac{dG}{dx}(x) dx. \quad (69)$$

(3) When the function f is monotone increasing in $[a, b]$ and the function G satisfies

$$\max_{y \in [a, b]} (G(b) - G(a)) = G(b) - G(a), \quad (70)$$

we have

$$\int_a^b f(x) \frac{dG}{dx}(x) dx \leq f(b) \int_a^b \frac{dG}{dx}(x) dx. \quad (71)$$

(4) When the function f is monotone decreasing in $[a, b]$ and the function G satisfies

$$\min_{y \in [a, b]} (G(b) - G(a)) = G(b) - G(a), \quad (72)$$

we have

$$\int_a^b f(x) \frac{dG}{dx}(x) dx \leq f(b) \int_a^b \frac{dG}{dx}(x) dx. \quad (73)$$

Proof of Lemma 12: We first show Item (1). We assume that G is C^1 -continuous. We assume that there are points $a = a_1, a_2, \dots, a_{2n} = b$ such that $g(x) \leq 0$ for $a_{2i} < x < a_{2i+1}$ and $g(x) \geq 0$ for $a_{2i+1} < x < a_{2i+2}$. So, the assumption of this lemma implies that $\int_{a_{2n-2}}^{a_{2n-1}} g(x) dx + \int_{a_{2n-1}}^{a_{2n}} g(x) dx \geq 0$. We choose a point a'_{2n+1} such that $\int_{a_{2n-2}}^{a_{2n-1}} g(x) dx + \int_{a'_{2n-1}}^{a_{2n}} g(x) dx = 0$. We define the function g_1 as

$$g_1(x) := \begin{cases} g(x) & \text{for } x < a_{2n-2} \\ 0 & \text{for } a_{2n-2} \leq x \leq a'_{2n-1} \\ g(x) & \text{for } a'_{2n-1} \leq x \end{cases} \quad (74)$$

Then, we have

$$\int_a^b f(x) g(x) dx \leq \int_a^b f(x) g_1(x) dx. \quad (75)$$

and $g_1(x) \geq 0$ for $a_{2n-3} < x < b$. Rewriting a_{2n} by a_{2n-2} , we repeat the above process for g_1 and denote the resultant function by g_2 . Repeating this procedure, we define g_1, g_2, \dots, g_n . So, g_n satisfies $g_n(x) \geq 0$ on (a, b) and

$$\int_a^b f(x) g(x) dx \leq \int_a^b f(x) g_n(x) dx. \quad (76)$$

Since

$$\int_a^b f(x) g_n(x) dx \leq f(a) \int_a^b g_n(x) dx, \quad (77)$$

we obtain the desired statement of Item (1) when G is C^1 -continuous. In the general case, G can be approximated by a C^1 -continuous function satisfying the desired conditions. So, we obtain the desired statement of Item (1) in the general case.

Applying $-f$ and $-g$ to Item (1), we obtain Item (2). Applying $f(a+b-x)$ and $g(a+b-x)$ to Items (1) and (2), we obtain Items (3) and (4), respectively. ■

Proof of Lemma 11: To show (62), it is enough to discuss the case when $f(0) = 0$ because the general case can be obtained by substituting $f(x) - f(0)$ into f . Also, it is enough to show that

$$\begin{aligned} & \left| \int_{-\infty}^{\infty} f(x) P_X(dx) - \int_{-\infty}^{\infty} f(x) P_{X'}(dx) \right| \\ & \leq 2 \sup_x (f(x) - f(0)) \sup_x |F_X(x) - F_{X'}(x)|. \end{aligned} \quad (78)$$

We choose

$$x_1 := \operatorname{argmax}_{x \in (-\infty, 0]} (F_X(x) - F_{X'}(x)) \quad (79)$$

$$x_2 := \operatorname{argmin}_{x \in [0, \infty)} (F_X(x) - F_{X'}(x)). \quad (80)$$

We choose R sufficiently large.

Items (1) and (4) of Lemma 12 imply

$$\int_{-R}^{x_1} f(x) P_X(dx) - \int_{-R}^{x_1} f(x) P_{X'}(dx) \leq f(-R)[(F_X(x_1) - F_X(-R)) - (F_{X'}(x_1) - F_{X'}(-R))] \quad (81)$$

$$\int_{x_1}^0 f(x) P_X(dx) - \int_{x_1}^0 f(x) P_{X'}(dx) \leq f(0)[(F_X(0) - F_X(x_1)) - (F_{X'}(0) - F_{X'}(x_1))] = 0, \quad (82)$$

respectively. Similarly, Items (2) and (3) of Lemma 12 imply

$$\int_{x_2}^R f(x) P_X(dx) - \int_{x_2}^R f(x) P_{X'}(dx) \leq f(R)[(F_X(R) - F_X(x_2)) - (F_{X'}(R) - F_{X'}(x_2))] \quad (83)$$

$$\int_0^{x_2} f(x) P_X(dx) - \int_0^{x_2} f(x) P_{X'}(dx) \leq f(0)[(F_X(x_2) - F_X(0)) - (F_{X'}(x_2) - F_{X'}(0))] = 0, \quad (84)$$

respectively. Combining them, we have

$$\begin{aligned} & \int_{-R}^R f(x) P_X(dx) - \int_{-R}^R f(x) P_{X'}(dx) \\ & \leq f(-R)[(F_X(x_1) - F_X(-R)) - (F_{X'}(x_1) - F_{X'}(-R))] \\ & \quad + f(R)[(F_X(R) - F_X(x_2)) - (F_{X'}(R) - F_{X'}(x_2))]. \end{aligned} \quad (85)$$

Taking the limit $R \rightarrow \infty$, we have

$$\begin{aligned} & \int_{-\infty}^{\infty} f(x) P_X(dx) - \int_{-\infty}^{\infty} f(x) P_{X'}(dx) \\ & \leq \lim_{R \rightarrow \infty} f(-R)[F_X(x_1) - F_{X'}(x_1)] + \lim_{R \rightarrow \infty} f(R)[F_{X'}(x_2) - F_X(x_2)], \end{aligned} \quad (86)$$

which implies (78). ■

X. DISCUSSION

We have proposed the noise injecting attack as a very strong attack to secure wireless communication, in which, Eve can control everything except for the neighborhood of Alice and Bob's detector. Under a reasonable assumption (A1)-(A4) for the performance of Eve's detector, i.e., under the model (1) and (2), we have constructed a secure key generation protocol by using backward reconciliation over the noise injecting attack. For this analysis, as Theorem 1, in the noise injecting attack we have shown that Eve's information can be reduced to the single random variable E' as Theorem 1.

Also, as Lemma 2, when the additive noise generated during the transmission is subject to a Gaussian distribution, we have derived a necessary and sufficient condition (7) of the coefficients a_B , a_E , and b_E and the variance of its Gaussian random variable for realizing greater correlation coefficient, i.e., greater mutual information between Alice and Bob than that between Bob and Eve under a spatial condition for Eve. Even when it is difficult to realize the condition (7), we have proposed the post selection method, in which, we choose only the case when the condition (7) holds by utilizing the stochastic behavior of the LHS of (7).

To identify the channel between Alice and Bob, our protocol contains the estimation of channel. In particular, we do not assume that the additive noise generated during transmission is a Gaussian random

variable. So, we need a non-parametric estimation, which has been resolved by Kolmogorov-Smirnov test [51], [52]. Combining a suitable exponential upper bound for the leaked information and the above error evaluation, we derived finite-length security analysis as Theorem 6. As Fig 3, we give a numerical calculation for the upper bound given in (32) in a typical example.

When Eve breaks the quasi static condition, she can change the artificial noise dependently of the pulse. In this case, if Alice and Bob pre-agree which pulses are used for samples, Eve can insert the large artificial noise only to the non-sampling pulses so that the condition (7) does not hold in the non-sampling pulses without detection by Alice and Bob. Then, Eve can succeed in eavesdropping without detection by Alice and Bob. Currently, we might not have such a technology, however, we cannot deny such an eavesdropping in future. Fortunately, in our protocol, Alice and Bob do not fix the sampling pulse priorly, they choose the sample pulse after the transmission from Alice to Bob as the random sampling, whose security guaranteed by authentication. Then, Eve cannot selectively insert the artificial noise. These effects are the same as those in BB84 protocol of quantum key distribution [25]. However, the errors in Bob's observations is not necessarily subject to an identical and independent distribution. The evaluation of estimation becomes more complicated. In the discrete variables case, the behavior of this random sampling can be discussed by hypergeometric distribution like quantum key distribution [56], [57]. Since our system employs the continuous variable, this type evaluation is not so easy.

ACKNOWLEDGMENTS

The author is very grateful to Professor Hideichi Sasaoka and Professor Hisato Iwai for helpful discussions and informing the references [9], [10], [11], [12]. He is also grateful to Professor Ángeles Vazquez-Castro, Professor Matthieu Bloch, Professor Shun Watanabe, Professor Himanshu Tyagi, and Dr. Toyohiro Tsurumaru for helpful discussions and comments. The works reported here were supported in part by a MEXT Grant-in-Aid for Scientific Research (B) No. 16KT0017, the Okawa Research Grant and Kayamori Foundation of Informational Science Advancement.

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